

On Complex-Valued Solutions to a 2D Eikonal Equation. Part Two: Existence Theorems

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Abstract

The following equation $w_x^2 + w_y^2 + n^2(x, y) = 0$, which arises in generalizations of geometrical optics, is investigated from a theoretical point of view. Here x and y denote rectangular coordinates in the Euclidean plane, n is real-valued and strictly positive. A framework is set up that involves a Bäcklund transformation relating $\operatorname{Re}(w)$ and $\operatorname{Im}(w)$, second-order PDEs in divergence and non-divergence form governing $\operatorname{Re}(w)$, a variational integral, free boundary problems, boundary value problems, viscosity solutions. The present paper is a continuation of a preceding one, [MT], where qualitative properties of smooth solutions are offered. Here the existence of solutions, which need not be smooth, is discussed.