

ON COMPLEX-VALUED SOLUTIONS to a  
2D EIKONAL EQUATION.  
PART ONE: QUALITATIVE PROPERTIES

Rolando Magnanini & Giorgio Talenti

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**Abstract**

$w_x^2 + w_y^2 + n^2(x, y) = 0$  is a two-dimensional version of the *eikonal equation* appearing in the generalizations of geometrical optics that deal with diffraction. Here  $x$  and  $y$  denote rectangular coordinates in the Euclidean plane, and  $n$  is real-valued. A framework is proposed, which consists of Bäcklund transformations and second-order PDEs governing  $\operatorname{Re}(w)$  and  $\operatorname{Im}(w)$ . Sample solutions are constructed in the case where  $n$  is constant. The critical points of  $\operatorname{Re}(w)$  are the main motif. Theorems, focusing on the geometry of such critical points, are given.