High-Order Time-Stepping for Nonlinear PDE through Rapid Estimation of Block Gaussian Quadrature Nodes

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The stiffness of systems of ODEs that arise from spatial discretization of PDEs causes difficulties for both explicit and implicit time-stepping methods. Krylov Subspace Spectral (KSS) methods present a balance between the efficiency of explicit methods and the stability of implicit methods by computing each Fourier coefficient from an individualized approximation of the solution operator of the PDE. While KSS methods are explicit methods that exhibit a high order of accuracy and stability similar to that of implicit methods, their efficiency needs to be improved. A previous asymptotic study of block Lanczos iteration yielded estimates of extremal block Gaussian quadrature nodes for each Fourier component and led to an improvement in efficiency. In this talk, a more detailed asymptotic study is performed in order to rapidly estimate *all* nodes, thus drastically reducing computational expense without sacrificing accuracy.

Exponential propagation iterative (EPI) methods provide an efficient approach to the solution of large stiff nonlinear systems of ODE, compared to standard integrators. However, the bulk of the computational effort in these methods is due to products of matrix functions and vectors, which can become very costly at high resolution due to an increase in the number of Krylov projection steps needed to maintain accuracy. In this talk, it is proposed to modify EPI methods by using KSS methods, instead of standard Krylov projection methods, to compute products of matrix functions and vectors. Numerical experiments demonstrate that this modification causes the number of Krylov projection steps to become bounded independently of the grid size, thus dramtically improving efficiency and scalability.