Boundary Value Methods for the reconstruction of Sturm-Liouville potentials.

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Abstract

An inverse Sturm-Liouville problem consists of recovering the potential $q(x) \in L^2(0, \pi)$ of the differential operator

$$L = -\frac{d^2}{dx^2} + q(x)$$

starting from the knowledge of suitable spectral data. The existence and uniqueness of its solution has been proved for several formulation of it.
Recently, a numerical procedure for the solution of the so-called *symmetric* inverse problem has been proposed in [3] which looks for a continuous approximation of the unknown potential belonging to suitable function spaces of finite dimension. In order to effectively compute such approximation a sequence of direct problems has to be solved. This is done by applying one of the Boundary Value Methods introduced in [1, 2] which generalize the classical Numerov scheme. The proposed procedure for the symmetric inverse problem has provided competitive results with respect to other techniques already available in the literature.
By virtue of this fact, the described approach has been extended for the definition of numerical methods for the *two-spectra* and the *half* inverse problems.
In this talk, all the mentioned procedures will be illustrated and some numerical results, confirming their effectiveness, will be presented.

References

[1] L. Aceto, P. Ghelardoni, C. Magherini. Boundary value methods as an extension of Numerov's method for Sturm-Liouville eigenvalue estimates, *Appl. Numer. Math.* **59** (2009), no. 7, 16441656.

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[3] P. Ghelardoni, C. Magherini. BVMs for computing Sturm-Liouville symmetric potentials,*Appl. Math. Comput.* **217** (2010), no. 7, 30323045.