Josef Dalík: Averaging of Gradients in Vertices of Triangulations

For a conforming shape-regular triangulation $\mathscr{T}_h$ without obtuse angles of a bounded polygonal domain $\Omega \subset \Re^2$, the so-called weighted averaging operator relating a high-order approximation $\mathrm{W}[z,\Pi_h(u)](a)$ of the directional derivative $\partial u/\partial z(a)$ to a unit vector z, inner or so-called semi-inner vertex a of $\mathscr{T}_h$ and to the interpolant $\Pi_h(u)$ of a smooth function u in the finite element space related to $\mathscr{T}_h$ and the linear triangular finite elements will be presented. This operator will be generalized to an averaging operator for the high-order approximations of partial derivatives of arbitrary orders of smooth functions in d variables for any $d \geq 2$, essential properties of W and its generalization will be described and their accuracy will be compared with the accuracies of other known operators of this kind numerically. Among other possible applications, these operators can be used in a posteriori error estimators of numerical solutions of the boundary-value problems for the partial differential equations, suitable to control their adaptive numerical solvers.