High order finite difference schemes for the numerical solution of eigenvalue problems for IVP-ODEs

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Abstract

In this talk we will investigate the numerical solution of eigenvalue problems

$$F(t) \equiv p(t)y''(t) + q(t)y'(t) + r(t)y(t) = \lambda w(t)y(t), \qquad \lambda \in \Re, \qquad (1)$$

$t \in [0,\ 1]$, with initial conditions $y(0) = y_0$ and $y'(0) = y'_0$ by means of high order finite difference schemes. Such problems arise from many practical applications in physics and engineering for example (signal processing) wave functions and Fourier series expansions. Some of these problems can be singular in the initial point.

The numerical method used is based on the idea initially proposed in [2] for boundary value problems and then generalized to initial value problems in [1]; that is, to approximate each derivative in (1) by means of appropiate different finite difference schemes. As outlined in [1], given a discretization $0 \equiv t_0 < t_1 < \ldots < t_n \equiv 1$ of $[0,\ 1]$, we compute an $n \times n$ linear system from $F(t_i) = 0$, $i = 1, \ldots, n$, by considering the known value y'_0 inside some of the initial schemes (for example, $y''(t_1)$ is approximated by means of a linear combination involving $y'_0, y_0, y_1, \ldots, y_k$).

Similarly to the most popular techniques called matrix methods, the eigenvalues of the coefficient matrix well approximate the first n values of the parameter λ. Better approximations of a specific eigenvalue and of the associated eigenfunction are obtained by solving the nonlinear problem for $y_1, \ldots, y_n, \lambda$

$$F(t_i) = \lambda w(t_i)y(t_i), \qquad i = 1, \ldots, n, \qquad (2)$$

in addition to a condition of normalization for the eigenfunction.

Theoretical and numerical results will be proposed in order to show the efficiency of the method as well as the accuracy of the computed eigenvalues and eigenfunctions for regular initial value problems and more difficult ones as singular problems.

References

[1] P. Amodio, G. Settanni, *High order finite difference schemes for the solution of second order initial value problems*, J. Numer. Anal., Industrial and Applied Mathematics (2010), in press.

[2] P. Amodio, I. Sgura, *High Order Finite Difference Schemes for the Solution of Second Order BVPs*, J. Comput. Appl. Math. **176** (2005), 59–76.